

FINAL EXAM ANSWERS
SM 223 Calculus III Fall 2001-02 7 Dec 1330-1630

1. D
2. B
3. B
4. A
5. C
6. A
7. E
8. A
9. C
10. D
11. A
12. D
13. D
14. D
15. A
16. E
17. C
18. A
19. B
20. E

FINAL EXAM ANSWERS
SM 223 Calculus III Fall 2001-02 7 Dec 1330-1630

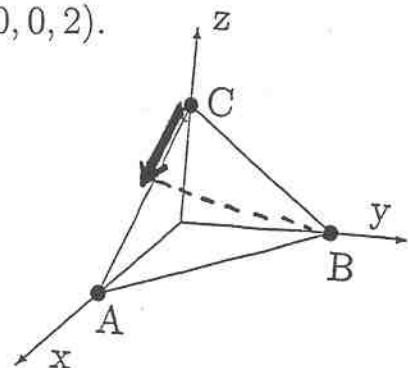
21. Consider the three points $A = (5, 0, 0)$, $B = (0, 3, 0)$, $C = (0, 0, 2)$.

(a) $\overrightarrow{CA} \cdot \overrightarrow{CB} = \langle 5, 0, -2 \rangle \cdot \langle 0, 3, -2 \rangle = 4$

(b) $\cos(\angle C) = \frac{\overrightarrow{CA} \cdot \overrightarrow{CB}}{|\overrightarrow{CA}| |\overrightarrow{CB}|} = \frac{4}{\sqrt{29}\sqrt{13}}$. So $\angle C \approx 78^\circ$

(c) $\text{proj}_{\overrightarrow{CA}} \overrightarrow{CB} = \left(\frac{\overrightarrow{CA} \cdot \overrightarrow{CB}}{\overrightarrow{CA} \cdot \overrightarrow{CA}} \right) \overrightarrow{CA} = \frac{4}{29} \langle 5, 0, -2 \rangle$

(d) See sketch.



22. (a) $\overrightarrow{CA} \times \overrightarrow{CB} = \langle 5, 0, -2 \rangle \times \langle 0, 3, -2 \rangle = \langle 6, 10, 15 \rangle$

(b) Area $\triangle ABC = \frac{1}{2} |\overrightarrow{CA} \times \overrightarrow{CB}| = \frac{1}{2} | \langle 6, 10, 15 \rangle | = \frac{19}{2}$

(c) Plane: $6x + 10y + 15(z - 2) = 0$ or $\frac{x}{5} + \frac{y}{3} + \frac{z}{2} = 1$ or $6x + 10y + 15z = 30$.

(d) Line: $x = 6t$, $y = 10t$, $z = 15t$, $(-\infty < t < \infty)$.

(e) We seek the intersection point of the line and the plane.

$$6(6t) + 10(10t) + 15(15t) = 30.$$

So $t = 30/361$. This gives the point $(x, y, z) = (180/361, 300/361, 450/361)$.

23. We seek to minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $g(x, y, z) = 2x + 3y + 3z = 66$.

Lagrange multipliers tells us to solve the system $\{\nabla f = \lambda \nabla g; g = 66\}$.

This gives the system

$$\{2x = \lambda \cdot 2, \quad 2y = \lambda \cdot 3, \quad 2z = \lambda \cdot 3, \quad 2x + 3y + 3z = 66\},$$

which leads to $\lambda = 6$, and $(x, y, z) = (6, 9, 9)$.

24. $\vec{r}(t) = \langle 6 \cos(t/2), 8 \sin(t/2) \rangle$

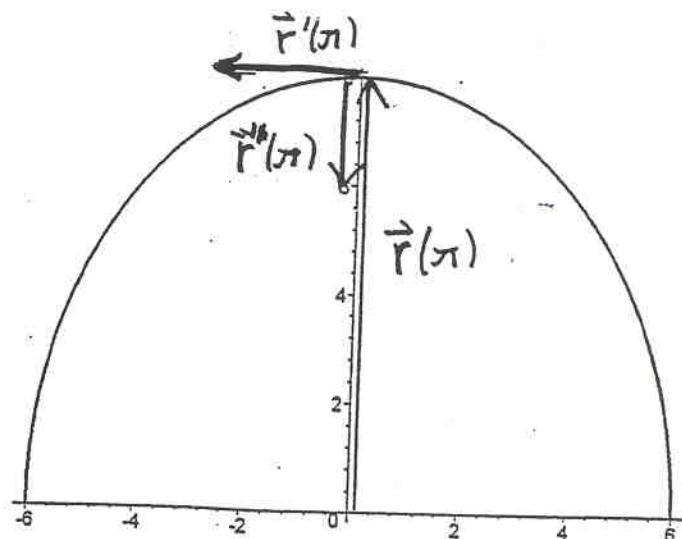
(a) $\vec{r}(\pi) = \langle 0, 8 \rangle$

(b) $\vec{r}'(\pi) = \langle -3, 0 \rangle$

(c) $\vec{r}''(\pi) = \langle 0, -2 \rangle$.

(d) Speed: $|\vec{r}'(\pi)| = 3$.

(e) See the sketch



FINAL EXAM ANSWERS
SM 223 Calculus III Fall 2001-02 7 Dec 1330-1630

25. The position vector is

$$\vec{r}(t) = \langle x(t), y(t) \rangle = \langle v_0 \cos(\alpha)t, -gt^2/2 + (v_0 \sin(\alpha)t + h_0) \rangle = \langle 50\sqrt{3}t, -5t^2 + 50t + 55 \rangle$$

(a) Solve $0 = y(t) = -5t^2 + 50t + 55$ and find that $t = 11.0$.

$$(b) |\vec{r}'(11.0)| = |\langle 50\sqrt{3}, -10 \cdot 11 + 50 \rangle| = \sqrt{11100} \approx 105.3 \text{ m/sec}$$

26. $P(K, L) = 8K^{1/2}L^{1/2}$. Currently $K = 100$ and $L = 25$.

(a) Current production: $P(100, 25) = 400$ tons/month.

(b) $P_L(100, 25) = 8$ means for every \$1000 more we spend on labor, production will increase by 8 tons/month (under current conditions).

(c) $P_K(100, 25) = 2$

(d) By the Chain Rule and the above computations

$$\frac{\partial P}{\partial t} = \frac{\partial P}{\partial K} \frac{\partial K}{\partial t} + \frac{\partial P}{\partial L} \frac{\partial L}{\partial t} = 2 \cdot 4 + 8 \cdot 3 = 32 \text{ tons/month}$$

27. (a) The sketch shows a contour map for a function $f(x, y)$ with the contours $f(x, y) = k$ drawn for $k = 0, 5, 10$. By the midpoint rule

$$\int_0^8 \int_0^{16} f(x, y) dy dx \approx (4 \cdot 8)(10 + 5 + 5 + 0) = 640$$

(b) Heat index $F(T, H)$; $F_T(96, 70) \approx 3.75$.

i. $F(96, 70) = 125$

ii. $F_H(96, 70) \approx 0.9$

iii. Linear Approximation: $F(T, H) \approx 125 + 3.75(T - 96) + 0.9(H - 70)$.

So $F(97, 72) \approx 130.55 \approx 131$

heat index F	$H = 60$	$H = 65$	$H = 70$	$H = 75$	$H = 80$
$T = 92$	105	108	112	115	119
$T = 94$	111	114	118	122	127
$T = 96$	116	121	125	130	135
$T = 98$	123	127	133	138	155

FINAL EXAM ANSWERS
SM 223 Calculus III Fall 2001-02 7 Dec 1330-1630

28. Let $f(x, y) = x^2 + y^2 + x^2y + 4$.

- (a) $\nabla f(x, y) = \langle 2x + 2xy, 2y + x^2 \rangle$
 - (b) $D_{\vec{u}} f(2, 3) = \nabla f(2, 3) \cdot \vec{u} = \langle 16, 10 \rangle \cdot \frac{1}{5}\langle 3, 4 \rangle = 88/5 = 17.6$
 - (c) Critical points: $(x, y) = (0, 0), (\sqrt{2}, -1), (-\sqrt{2}, -1)$
 - (d) $(0, 0)$: relative minimum;
 $(\pm\sqrt{2}, -1)$: saddle points.
-

29. (a) Tetrahedron bounded by planes $x = 0, y = 0, z = 0, 3x + 2y + z = 6$.

$$\text{Volume} = \int_0^{\frac{3}{2}} \int_0^{\frac{(6-2y)/3}{2}} \frac{6 - 2y - 3x}{2} dx dy$$

(b)

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) dz dy dx = \int_0^{\frac{1}{2}} \int_0^{\frac{1-z}{2}} \int_0^{\frac{y^2}{2}} f(x, y, z) dx dy dz$$

(c)

$$\text{Volume} = \int_0^{2\pi} \int_0^{\frac{2}{r}} \frac{(4 - r^2) r}{2} dr d\theta$$

30. Given iterated integral: $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} (x^2 + y^2 + z^2) dz dy dx$

(a) Spherical coordinates

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^2 \rho^2 \cdot \rho^2 \sin(\phi) d\rho d\phi d\theta$$

(b) Cylindrical coordinates

$$\int_0^{\pi/2} \int_0^2 \int_0^{\sqrt{4-r^2}} (r^2 + z^2) r dz dr d\theta$$

(c) Evaluate: $16\pi/5$.
